Bohr's Atomic Model

Postulates of Bohr's atomic model

i) **First Postulate**: Electron can revolve round the nucleus only in those permissible orbits for which the angular momentum of the electron is equal to integral multiple of $h/2\pi$.

If m is mass and v be velocity of an electron revolving in an orbit of radius r then according to Bohr's postulate,

$$\text{Angular momentum (L)} = \frac{nh}{2\pi}$$

$$\text{or,} \quad mvr = \frac{nh}{2\pi}$$

Where, n = 1, 2, 3, ……. is called principal quantum number.

ii) **Second Postulate**: Electron emits energy when it jumps from higher energy states to lower energy states and it absorbs energy when it jumps from lower to higher energy states.

When an electron jumps from higher energy state En_2 to lower energy state En_1 then the energy radiated is,

$$E = En_2 - En_1$$

$$\text{Or, } hf = En_2 - En_1$$

Bohr's theory of Hydrogen atom

Radius of n^th^ orbit of Hydrogen atom

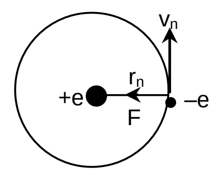


Fig: Electron revolving round the nucleus

Hydrogen atom consist a nucleus having positive charge +e. Let m be the mass and –e be the charge of an electron revolving around the nucleus in n^{th} stationary orbit of radius r_n with orbital velocity v_n as shown in Fig.

The electrostatic force of attraction between nucleus (proton) and electron is given by

$$F_e = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r_n^2} \quad \text{……………..(i)}$$

Where ε_o is permittivity of free space.

The centripetal force acting on the electron in the circular orbit is given by

$$F_c = \frac{mv_n^2}{r_n} \quad \text{..................(ii)}$$

Here, the electrostatic force of attraction between nucleus and electron provides necessary centripetal force for the electron. Therefore,

$$F_e = F_c$$

$$\text{or,} \quad \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r_n^2} = \frac{mv_n^2}{r_n}$$

$$\text{or, } v_n^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n} \quad \text{…………………(iii)}$$

Form Bohr's first postulate,

$$mv_n r_n = \frac{nh}{2\pi}$$

$$\text{or, } v_n = \frac{nh}{2\pi m r_n} \text{..................(iv)}$$

Where, $h = 6.626 \times 10^{-34}$ Js is Planck's constant.

From equations (iii) and (iv), we get

$$\left(\frac{nh}{2\pi m r_n}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n}$$

$$\frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{e^2}{4\pi\varepsilon_0 m r_n}$$

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$$

This equation gives the radius of n^{th} orbit of hydrogen atom. From the above equation, it is clear that the radius is directly proportional to n^2 (i.e. $r_n \propto n^2$).

Bohr's radius

The radius of inner most orbit of hydrogen atom is called Bohr's radius. It is denoted by a_0.

For Bohr's radius, n =1

$$\therefore \quad r_1 = a_0 = \frac{\varepsilon_o h^2}{\pi m e^2}$$

Substituting the known values of ε_0, h, m and e, we have

Bohr radius (a_o) = $\frac{\varepsilon_o h^2}{\pi m e^2}$ = 0.529Å = 0.53 Å

From this relation we can find the radii of the different orbits of hydrogen atom

Velocity of electron in the n^{th} orbit of hydrogen atom

From the Bohr's postulate,

$$m v_n r_n = \frac{nh}{2\pi}$$

$$\text{or, } v_n = \frac{nh}{2\pi m r_n}$$

$$\text{or, } v_n = \frac{nh}{2\pi m} \frac{\pi m e^2}{\varepsilon_o n^2 h^2}$$

$$\text{So, } v_n = \frac{e^2}{2\varepsilon_o nh}$$

This equation represents the velocity of an electron in n^{th} orbit of hydrogen atom.

Energy of electron in the n^{th} orbit of hydrogen atom

An electron revolving around the nucleus possesses kinetic energy as well as potential energy. The kinetic energy of electron is due to its motion whereas the potential energy is due to the electrostatic force of attraction between electron and nucleus. Thus, the total energy of an electron revolving round the nucleus is the sum of its kinetic and potential energy.

The kinetic energy of electron in n^{th} orbit is,

$$\text{K.E.} = \frac{1}{2} m v_n^2$$

$$\text{We have, } v_n = \frac{e^2}{2\varepsilon_o nh}$$

$$\text{So , K.E.} = \frac{1}{2} m \left(\frac{e^2}{2\varepsilon_o nh}\right)^2$$

$$\text{or, K. E.} = \frac{me^4}{8\varepsilon_o^2 n^2 h^2} \text{..............(i)}$$

The potential energy of electron in n^{th} orbit is given by

$$\text{P.E.} = \text{(electrostatic potential)} \times \text{(Charge of an electron)}$$

$$= \left(\frac{e}{4\pi\varepsilon_o r_n}\right) (-e)$$

$$\text{or,} \quad \text{P.E.} = \frac{-e^2}{4\pi\varepsilon_o r_n}$$

Substituting the value of r_n in above equation, we get

$$\text{P.E.} = \frac{-e^2}{4\pi\varepsilon_o} \frac{\pi me^2}{\varepsilon_o n^2 h^2}$$

$$\text{P.E.} = \frac{-me^4}{4\varepsilon_o^2 n^2 h^2} \text{.....................(ii)}$$

Now, the total energy of an electron in the n^{th} orbit is

$$E_n = \text{K. E + P.E.}$$

$$\text{or,} \quad E_n = \frac{me^4}{8\varepsilon_o^2 n^2 h^2} - \frac{me^4}{4\varepsilon_o^2 n^2 h^2}$$

$$\therefore \quad E_n = -\frac{me^4}{8\varepsilon_o^2 n^2 h^2}$$

The negative sign shows that the electron is bound to the nucleus. As the value of n increases, E_n also increases and vice-versa. Hence the outer orbits have more energy than the inner one.

Hydrogen Spectrum

When an electron in hydrogen atom jumps from higher energy state n_2 of energy En_2 to lower energy state n_1 of energy En_1 then the energy is radiated. The frequency of emitted radiation (i.e. photon) is given by

$$hf = E_{n_2} - E_{n_1} \text{ -------------- (i)}$$

As we know,

$$En_2 = \frac{-me^4}{8\varepsilon_o^2 n_2^2 h^2} \quad \text{and} \quad En_1 = \frac{-me^4}{8\varepsilon_o^2 n_1^2 h_2}$$

Then equation (i) becomes,

$$\text{or, } hf = \frac{-me^4}{8\varepsilon_o^2 n_2^2 h^2} - \left(\frac{-me^4}{8\varepsilon_o^2 n_1^2 h^2}\right)$$

$$= \frac{me^4}{8\varepsilon_o^2 n_1^2 h^2} - \frac{me^4}{8\varepsilon_o^2 n_2^2 h^2}$$

$$= \frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or, } f = \frac{me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Wave number of a radiation is the number of complete waves in unit length in vacuum. It is the reciprocal of wave length of radiation. It is denoted by $\bar{f}$.

$$\therefore \quad \text{Wave number } (\bar{f}) = \frac{1}{\lambda}$$

$$\text{or} \quad \bar{f} = \frac{1}{c/f}$$

$$\text{or,} \quad \bar{f} = \frac{f}{c}$$

$$\text{or,} \quad \frac{1}{\lambda} = \frac{f}{c}$$

$$\text{or,} \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where, $R = \frac{me^4}{8\varepsilon_o^2 ch^3} = 1.097 \times 10^7 \text{ m}^{-1}$ is Rydberg's constant. This relation is called Rydberg formula which is best applied only to hydrogen and hydrogen like atoms such as He, Li, Be etc.

Energy levels in hydrogen atom

The energy of an electron in different stationary orbits can be represented by horizontal lines, which is called energy levels and the diagram of such energy levels is called energy level diagram which is shown in Fig.

The energy of an electron revolving round the nucleus in n^{th} stationary orbit in hydrogen atom is given by,

$$E_n = -\frac{me^4}{8\varepsilon_o^2 n^2h^2}$$

Substituting values of m, e, ε_o and h in above equation, we get

$$E_n = -\frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times n^2 \times (6.626 \times 10^{-34})^2}$$

$$= \frac{-21.76 \times 10^{-19}}{n^2} \text{ J}$$

$$\therefore E_n = -\frac{13.6}{n^2} \text{ eV}$$

This expression represents the energy of an electron in n^{th} orbit of hydrogen and negative sign signifies that the electron is bounded to the nucleus.

For the first orbital hydrogen atom, n =1

$$E_1 = \frac{-13.6}{1^2} \text{ eV} = -13.6 \text{ eV}$$

It is the ground state energy of hydrogen atom.

For n = 2,

$$E_2 = \frac{-13.6}{2^2} \text{ eV} = -3.4 \text{ eV}$$

It is the energy of first excited state or second energy state of hydrogen atom.

Similarly, Energy of second and third excited state can be given respectively as,

$$E_3 = \frac{-13.6}{3^2}\ \text{eV} = -\ 1.5\ \text{eV}$$

$$E_4 = \frac{-\ 13.6}{4^2}\ \text{eV} = -\ 0.85\ \text{eV}$$

When n = ∞, then

$E_\infty = \frac{-13.6}{\infty^2}$ = 0; which is maximum. This state is called ionized state. In this state, the electron becomes free. The energy level diagram of hydrogen atom is shown in figure below:

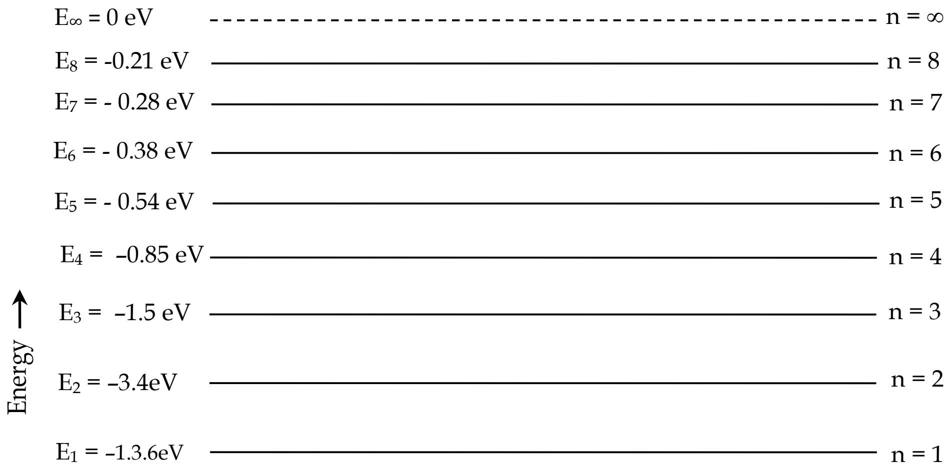


Fig : *Energy level diagram of hydrogen atom*

Spectral Series of Hydrogen Atom

When an electron in a hydrogen atom jumps from higher energy state to a lower energy state, it emits radiation of certain frequency or wave length which is called as spectral lines. The wavelength of spectral line depends on the two energy states between which transition of electron occurs. The transitions of electron between various orbits with different wavelengths produce the spectral series of hydrogen atom. The different types of spectral series of hydrogen atom are shown in Fig.

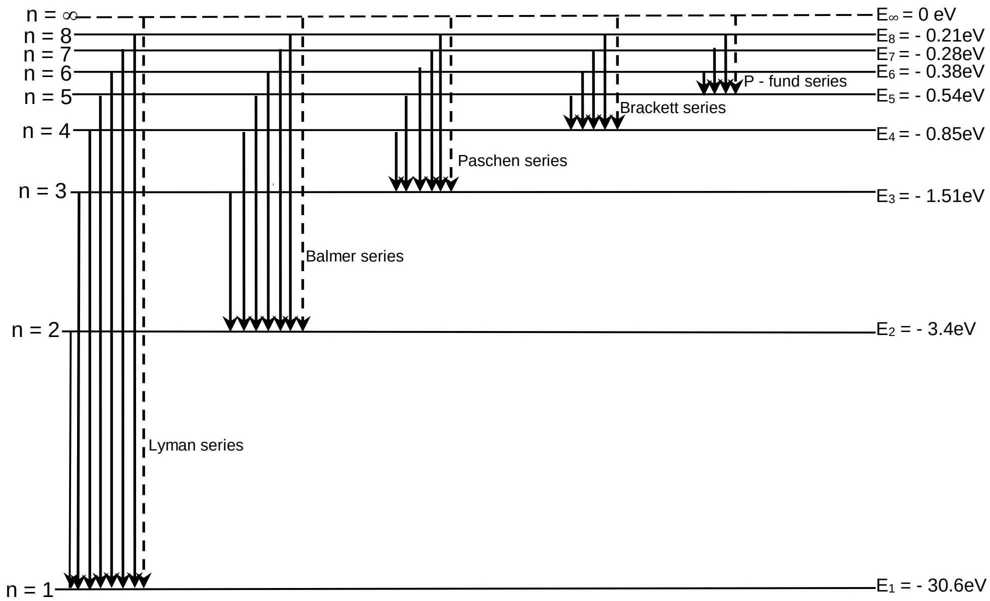


Fig- *Spectral series of hydrogen atom*

I) Lyman Series:

The spectral series obtained by the transition of electron from higher energy states to the lowest energy sate i.e. ground state (n = 1) is called *Lyman series*. This series lies in *ultraviolet region*.

The wave lengths of Lyman series are given by

$$\bar{f} = \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, $n_1 = 1$ and $n_2 = 2, 3, 4 \ldots$ and $R = 1.097 \times 10^7\ m^{-1}$ is Rydberg's constant

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$= R\left(1 - \frac{1}{n_2^2}\right)$$

II) Balmer Series:

The spectral series obtained the transition of electron from higher energy states to the first excited state (n=2) is called *Balmer series.* This series lies in *visible* region.

The wave lengths of Balmer series are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, $n_1 = 2$ and $n_2 = 3, 4, 5 \ldots$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

III) Paschen Series

The spectral series obtained by the transition of electron from higher energy states to the second excited state (n=3) is called *Paschen series.* This series lies in *infrared region*.

The wave lengths of Paschen series are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, $n_1 = 3$ and $n_2 = 4,5,6 \ldots$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^{\,2}}\right)$$

IV) Brackett Series:

The spectral series obtained by the transition of electron from higher energy states to the third excited state (n=4) is called *Brackett series.* This series lies in *infrared region*.

The wave lengths of Brackett series are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right)$$

Here, $n_1 = 4$ and $n_2 = 5, 6, 7$……

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^{\,2}}\right)$$

v) P-fund series

The spectral series obtained by the transition of electron from higher energy states to the forth excited state (n=5) is called *P- fund series.* This series lies in for infrared region.

The wave lengths of P-fund series are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right)$$

Here, $n_1 = 5$, and $n_2 = 6, 7, 8$….

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_2^{\,2}}\right)$$

Limitations of Bohr's theory of hydrogen atom:

i) It can explain only the spectra of hydrogen like simple atoms but it cannot explain the spectra of the atoms having many electrons.

ii) It explains only about the circular orbits but the elliptical orbits are also possible.

i) The fine structure of the spectral lines cannot be explained on the basis of this theory.

ii) It fails to explain Zeeman effect and stark effect.

v) It cannot explain about the intensity of spectral lines.

Excitation energy and Excitation potential

The minimum amount of energy required for the transition of an electron from its ground state to any excited state is called exciting energy.

The amount of energy required to excite an electron form ground state (i.e. n = 1) to its first excitation state (i.e. n =2) is called first excitation energy. The energy required to excite an electron from ground state to first excite state in hydrogen atom is given by

$$\begin{aligned} E &= E_2 - E_1 \\ &= -3.4 - (-13.6) \\ &= 10.2 \text{ eV.} \end{aligned}$$

Similarly, the energy required to excite the electron to second excited state in hydrogen atom is,

$$\begin{aligned} E &= E_3 - E_1 \\ &= -1.51 - (-13.6) \\ &= 12.09 \text{ eV} \end{aligned}$$

The minimum potential required to excite an electron from its ground state to the given excited state is called excitation potential. e.g. the excitation potential for the first excited state of H-atom is 10.2 V.

Ionization energy and ionization potential

The minimum amount of energy required to ionize an atom is called as *ionization energy* hence the minimum amount of energy required to make the electron free from the atom is the ionization energy.

The energy required to excite an electron from ground state to ionized state is given by,

$$E = E_\infty - E_1$$
$$= 0 - (-13.6)$$
$$= 13.6 \text{ eV}$$

The ionization energy is numerically equal to ground state energy

Hence, ***the minimum potential required to remove the electron from ground state to ionized state is called ionization potential***. The ionization potential is 13.6 V for hydrogen atom.

de-Broglie's Theory: Duality

According to de-Broglie; a moving particle behaves sometimes as a wave and sometimes as a particle. So, every moving particle is associated with wave. The waves associated with a moving particle are called *matter waves* or *de-Broglie waves*. The wavelength associated with matter waves is called *de-Broglie wave length*.

The de Broglie waves length of a particle of mass (m) moving with speed (v) is

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

Where h is plank's constant and p = mv is linear momentum of a particle.

The de Broglie wavelength is independent of nature and charge of the particle.

Derivation of de- Broglie's wavelength

Consider a photon to be a particle of mass 'm' moving with velocity of light 'c'. According to Einstein's mass energy relation; the energy of photon is,

$$E = mc^2 \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

According to Planck's quantum theory of radiation, the energy of photon having frequency f is given by

$$E = hf \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Where, h is planck's constant.

Comparing equations (i) and (ii), we get

$$hf = mc^2$$

$$\text{or,} \quad h.\frac{c}{\lambda} = mc^2$$

$$\text{or,} \ \lambda = \frac{h}{mc} = \frac{h}{P} \ldots\ldots\ldots\text{(iii)}$$

Here, P = mc is momentum of photon

Since, the matter also possesses dual nature. So, the wavelength of the wave associated with a particle of mass 'm' moving with velocity 'v' is

$$\lambda = \frac{h}{P} = \frac{h}{mv} \ldots\ldots\text{(iv)}$$

This equation represents the expression for de-Broglie wavelength.

Heisenberg's Uncertainty Principle

This principles states that, ***it is impossible to determine precisely and simultaneously the values of both the members of pair of physical variables (or canonically conjugate variables) which describe the motion of an atomic system.*** According to Heisenberg, the product of uncertainties of such variables is greater or equal to $\frac{h}{2\pi}$, where h is Panck's constant.

According to this principle, the position and momentum of a particle cannot be determined simultaneously to any desired degree of accuracy.

If ΔP and ΔX be uncertainties in momentum and position respectively, then

$$\Delta P \,.\, \Delta X \geq \frac{h}{2\pi}$$

If Δx is very small (i.e. $\Delta x \rightarrow 0$) then Δp will be very large (i.e. $\Delta p \rightarrow \infty$) and vice versa. It means if one variable is measured accurately, the measurement of other quantity becomes uncertain.
The uncertainty relation for energy and time is

$$\Delta E \,.\, \Delta t \geq \frac{h}{2\pi}$$

Where, ΔE is uncertainty in energy and Δt is uncertainty in time.

X-rays

X-rays are electromagnetic waves of very short wave length. the wave length of x-rays lies between 10^{-12} m to 10^{-9} m.

Production of x-rays: Coolidge Tube

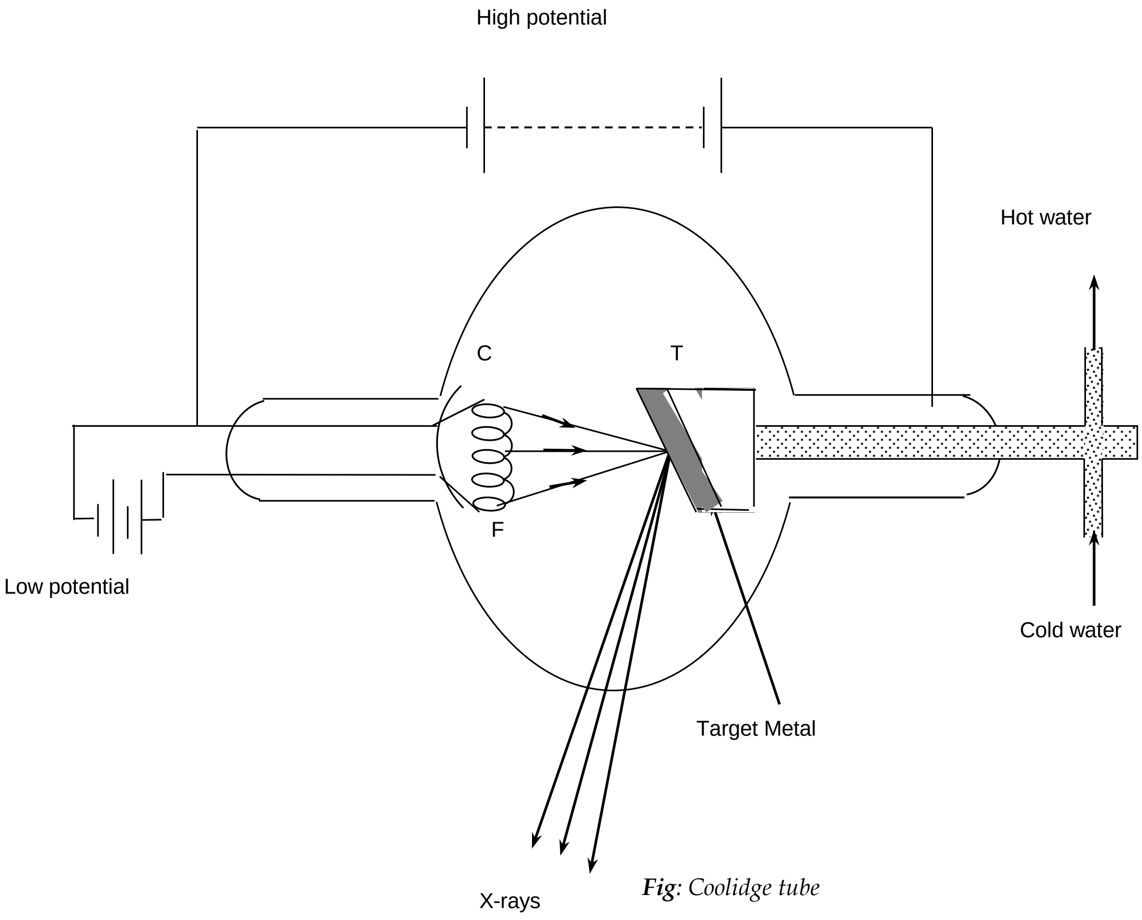


***Fig**: Coolidge tube*

X-rays can be produces by bombardment of fast moving electron i.e cathode rays on the metal target of high atomic weight and melting point.

X- rays are produce in Coolidge tube. It consists of an evacuated glass tube of pressure about 10^{-5} mm of Hg having cathode and anode. The cathode consists of tungsten filament (F) which is heated by low tension battery and is placed in a metal cap (C) to focus the electrons to the target. The metal target (T) i.e anode of tungsten or molybdenum having a high meting point and high atomic weight is kept at an angle of 45^0 to the horizontal. The target is connected the positive and the filament to negative terminal of high tension source. The cooling system is arranged to the target.

The filament F is heated by low tension battery and the electrons are emitted from it. These electrons are accelerated by applying very high potential (50-100 kV) between filament and target. These energetic electrons are focused to a point on the target with the help of metal cap (C). When the fast moving electrons strike the target, x-rays are produced. While producing x- rays, about 98% of the energy of the incident electrons is converted into heat and hence the target gets heated. So the target is cooled by circulating cold water through the copper pipe continuously.

Control of intensity and penetrating power

i. Control of intensity

The intensity of X- rays depends on the number of electrons striking the target metal. The number of electrons depends on the current passing through the filament.

i.e intensity of x-rays (I) $\propto$ filament current (I).

Hence the intensity of x-rays can be control by adjusting filament or, cathode current.

ii. Control of penetrating power

The **penetrating power of** the X –ray,which reptesents it's quality.it depends on its energy or frequency. The frequency of x-rays on depends on the voltage between filament and target.

Let m be the mass of an electron, e be the charge of an electron and V be the potential difference between two filament and target . Then,

Maximum kinetic energy gained = Work done

$$\frac{1}{2} mv_{max}^2 = eV,$$

Where, v_{max} is the maximum velocity of the electron striking on the target.

When whole of the kinetic energy of electron is converted into x-ray, then

Maximum energy of x-rays = Maximum kinetic energy of electron

$$\text{i.e } hf_{max} = \frac{1}{2} mv_{max}^2$$

Where, f_{max} is maximum frequency of x-rays and h is Planck's constant

From above equations, We have $hf_{max} = eV \quad \Rightarrow \quad f_{max} = \frac{eV}{h}$

Properties of X - rays

The properties of X-rays are as follows:

i) X-rays are electromagnetic radiations of short wavelength ranging from 10^{-12} m to 10^{-9}m.

ii) They are not deflected by electric as well as magnetic fields and hence they do not possess any charge.

iii) They travel with velocity of light in vacuum i.e 3×10^8 m/s.

iv) They affect photographic plates.

v) X- rays have ionizing power. They can ionize the gas, atom or matter through which they pass.

vi) They can produce photoelectric effect and Compton effect.

vii) They are highly energetic radiations so, they have high penetrating power. They can pass though opaque solids like wood, paper, flesh, thin sheet of metals etc.

Uses of X-rays

X-rays are widely used in various fields. Some important applications of x-rays are given below:

1. **Diagnosis:**
2. **Radio therapy**
3. **Detective departments:**
4. **Engineering:**
5. **Scientific research:**
6. **Industry**

<u>**Short answer questions**</u>

1) ***What are the limitations of Rutherford atomic model?***

   Ans. The limitations of Rutherford atomic model are:

   (i) It could not explain the stability of an atom.

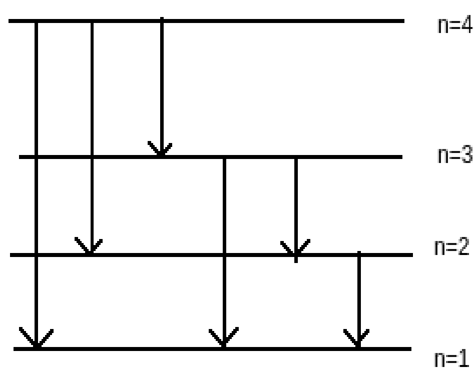
   (ii) It could not explain the origin of line spectra of hydrogen atom.

2) ***The total energy of an electron of an atom in an orbit is negative". What does this negative energy indicate?***

   Ans. The negative energy of electron in an orbit shows that the electron is bound to the nucleus. Greater the value of negative energy, more tightly the electron is bound to the nucleus. It shows that the energy must be supplied to escape the electron from the atom.

3) ***An electron is in the third excited state. How many photon wavelengths are possible?***

   Ans. If the electron is in the third excited state, then the six different form of transition and wavelengths are possible as shown in figure below.



4) ***Even if a hydrogen atom contains an electron, its spectrum consists of a large number of lines. Explain how. (Hint: same as above)***

5) ***An electron and proton have the same kinetic energy. Which one of them has the longer De – Broglie wavelength?***

   Ans. The De – Broglie wavelength is given by,

$$\lambda = \frac{h}{\sqrt{2mK.E.}}$$

For equal kinetic energy,

$\lambda \propto \frac{1}{\sqrt{m}}$

The mass of electron is less than the mass of proton i.e. $m_e < m_p$

So, the De – Broglie wavelength of electron is greater than that of proton.

6) ***The accelerating voltage of a proton is increased twice. How will its de Broglie wavelength change? Explain.***

Ans. The De – Broglie wavelength of proton is given by,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

If the accelerating voltage is increased by two times then the De – Broglie wavelength becomes,

$$\lambda' = \frac{h}{\sqrt{2me2V}} = \frac{1}{\sqrt{2}} \times \frac{h}{\sqrt{2meV}} = \frac{1}{\sqrt{2}} \lambda$$

7) ***The wave nature of particles is not observable in daily life. Why?***

Ans. The De – Broglie wavelength is given by,

$$\lambda = \frac{h}{mv}$$

This relation shows that the De – Broglie wavelength is inversely proportional to the mass of the particle. i.e. $\lambda \propto \frac{1}{m}$

In our daily life, we observe the particle having very large mass. As a result, wavelength becomes very small which can not be observable.

8) ***A stone is dropped from the top of a building. How does its de Broglie wavelength change?***

Ans. The De – Broglie wavelength of particle is given by,

$$\lambda = \frac{h}{mv}$$

where, m is mass and v is velocity of particle.

This relation shows that the De – Broglie wavelength is inversely proportional to the velocity of the particle.

When a stone is dropped from the top of the building, its velocity starts increasing due to gravity. As a result, its De – Broglie wavelength starts decreasing.

9) ***What are the differences between matter wave and electromagnetic wave?***

Ans. **Matter wave**

(i) These waves are associated with the particle.
(ii) These waves require medium to propagate.
(iii) Matter waves travel with different speed.
(iv) Speed of matter wave is less than the speed of light.

Electromagnetic wave

(i) These waves are associated with the photon.
(ii) These waves do not require medium to travel.
(iii) Electromagnetic waves travel with same speed.
(iv) Speed of electromagnetic wave is equal to the speed of light.

Numerical problems

Formula

- Angular momentum, $L = mvr = \frac{nh}{2\pi}$
- Energy of photon (transition between different orbits), $E = hf = \frac{hc}{\lambda} = E_2 - E_1$
- Radius of electron in the n^{th} orbit, $r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$
- Energy of electron in the n^{th} orbit, $E = -\frac{me^4}{8\varepsilon_o^2 n^2 h^2}$
- Energy of electron in the n^{th} orbit, $E = -\frac{13.6}{n^2}$

- Velocity of electron in the n^{th} orbit, $v_n = \frac{e^2}{2\varepsilon_o nh}$
- $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ Where, $R = \frac{me^4}{8\varepsilon_0{}^2ch^3}$
    - $R = 1.097 \times 10^7\ m^{-1}$ is Rydberg constant
    - $n_1<n_2$
    - For first line of lyman series, $n_1 = 1$ and $n_2 = 2$
    - For second line of lyman series, $n_1 = 1$ and $n_2 = 3$
    - for first line of Balmer series, $n_1 = 2$ and $n_2 = 3$
    - For second line of Balmer series, $n_1 = 2$ and $n_2 = 4$
    - For shortest wavelength, always take $n_2 = \alpha$ (infinity)
- De – Broglie wavelength:
    - $\lambda = \frac{h}{P} = \frac{h}{mv}$ (P is momentum)
    - $\lambda = \frac{h}{\sqrt{2\ meV}}$
    - $\lambda = \frac{h}{\sqrt{2mqV}}$ (For the charge q)
    - $\lambda = \frac{h}{\sqrt{2mK.E.}}$

    Note: $1\ eV = 1.6 \times 10^{-19} J$
- Uncertainty principle:

$$\Delta x.\Delta P = \frac{h}{2\pi}$$

$$\Delta x.m\Delta v = \frac{h}{2\pi}$$

Where, Δx is position accuracy and ΔP is momentum accuracy.

- Laser:

To calculate the number of photons striking per second,

$$\text{Power, } P = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{\lambda t}$$

$$n/t = \frac{p\lambda}{hc}$$

<u>Questions</u>

1) ***Find the wavelength of the radiation emitted from a hydrogen atom when an electron jumps from 4^{th} orbit to the second orbit, (given $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$ $h = 6.62 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg) (where the symbols have their usual meanings.)***

Solution: Given, $n_1 = 2$

$n_2 = 4$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$

$h = 6.62 \times 10^{-34}$ Js

$m_e = 9.1 \times 10^{-31}$ kg

$\lambda = ?$

We have, $R = \frac{me^4}{8\varepsilon_0{}^2ch^3}$ = ………… = $1.097 \times 10^7 m^{-1}$

Again,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\lambda = 4.89 \times 10^{-7}$ m

2) ***A hydrogen atom is in ground state. What is the quantum number to which it will be excited absorbing a photon of energy 12.75eV?***

Solution: Given,

$E = 12.75$eV

$E_1 = -13.6$ eV

$E_2 = ?$

We have,

$E = E_2 - E_1$

Or, $12.75 = E_2 - (-13.6)$

Or, $E_2 = 12.75 - 13.6 = -0.85$eV

Now,

$E_2 = \frac{-13.6}{n^2}$

Or, $-0.85 = \frac{-13.6}{n^2}$

Or, $n^2 = 16$

So, $n = 4$

3) ***The first member of Balmer series of hydrogen atom has a wavelength of 6563 Å. Calculate the wavelength of its second member.***

Solution:

For first line of Balmer series,

$n_1 = 2$

$n_2 = 3$

$\lambda = 6563$ Å $= 6563 \times 10^{-10}$m

R = ?

We have,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Or, $R = 1.097 \times 10^{7}\text{m}^{-1}$

For second line of Balmer series,

$n_1 = 2$

$n_1 = 4$

R = 1.097 x 107m-1

$\lambda = ?$

we have,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

…………………………….

λ =4.86 x 10^{-7}m

4) ***Calculate the wave length of electromagnetic radiation emitted by a hydrogen atom which undergoes a transition between energy levels of -1.36×10^-19 J and – 5.45x10^-19 J. (Given plank constant = 6.6×10^-34 Js).***
Solution:

E_1 = - $5.45x10^{-19}$ J

E_2 = -1.36×10^{-19} J

We have,

$E = E_2 - E_1$

Or, $\frac{hc}{\lambda} = -1.36\times10^{-19} - (-5.45x10^{-19})$

λ = 4.84 x 10^{-7} m

5) ***Calculate the De – Broglie wavelength of electron having K.E. 400 eV.***
Solution:

Given,

K.E. = 400 eV = 400 x 1.6 x 10^{-19}J

h = 6.62 x 10^{-34} JS

λ = ?

We have,

$\lambda = \frac{h}{\sqrt{2mK.E.}}$

= ………

= 6.13 x 10^{-11}m

6) ***A cricket ball is moving with a speed of 120 km/hr. What would be its de-Broglie wavelength if its mass is 400gms.***
Solution:

Given,

v = 120 km/hr = $\frac{120 \, x \, 1000}{60 \, x \, 60}$ = 33.33 km/hr

m = 400 gm = 0.4 kg

λ = ?

We have,

$\lambda = \frac{h}{mv}$ = ………. = 4.96 x 10^{-35}m

7) ***Determine the wavelength of a proton that has been accelerated through a potential difference of 20 kV. Mass of proton = 1.67×10^–27 kg and Planck constant = 6.62×10^–34 Js.***
Solution: Given,

P.d. (V) = 20 kV = 20000V

m = 1.67×10^{-27} kg

h = 6.62×10^{-34} Js

e = 1.6 x 10^{-19}J

λ = ?

We have,

$\lambda = \frac{h}{\sqrt{2meV}}$ = ……….. = 2.02 x 10^{-19}m

8) ***If an electron position can be measured to an accuracy of 10^{-9} m. How accurately can its velocity be measured? (m_e = 9.1 x 10^{-31} kg)***

Solution: Given,

$\Delta x = 10^{-9}$ m

m = 9.1 x 10^{-31}kg

Δv = ?

We have,

$\Delta x.\Delta P = \frac{h}{2\pi}$

Or, $\Delta x.\ m\Delta v = \frac{h}{2\pi}$

Or, Δv = ... = 1.16 x 10^{5}m/s

Short questions

1. Which has more energy, a photon in the infrared or photon in the ultraviolet? Give reasons.
2. An electron and proton have the same kinetic energy. Which one of them has the longer wavelength?
3. A photon and an electron have got the same de-Broglie wavelength. Which one has greater total energy? Explain.
4. The accelerating voltage of a proton is increased to twice. How will its de Broglie wavelength change? Explain.
5. Can X-ray diffraction experiment be performed by an ordinary grating? Why?
6. Explain the difference between stimulated and spontaneous emissions of radiation.
7. If a proton and an electron have the same speed, which has the longer de Broglie wavelength? Explain.
8. Even if a hydrogen atom contains an electron, its spectrum consists of a large number of lines. Explain how.
9. What is optical pumping in the production of laser?
10. Production of X-ray is the inverse phenomenon of photoelectric effect. Justify it.
11. "The total energy of an electron of an atom in an orbit is negative". What does this negative energy indicate?
12. An electron and proton are accelerated through the same potential, which one has higher- De-Broglie's wavelength? Justify your answer.
13. An electron and a proton have same kinetic energy, which of the two has greater de-Broglie wavelength Justify your answer.

14. A proton and an electron have same de-Broglie wavelengths, which of the two has greater kinetic energy? Justify your answer.
15. Distinguish between stimulated emission and spontaneous emission.
16. If a proton and an electron have the same kinetic energy which has the longer de Broglie wavelength?
17. Why is the gravitational force not taken into consideration while evaluating the energy of an electron in an atom?
18. Can X-rays be produced from gases? Explain.
19. A proton and an electron have the same speed. Which has longer wavelength?
20. The wave nature of particles is not observable in daily life. Why?
21. Can Bragg's law of x-ray diffraction be verified with yellow light of wavelength 600 nm? Explain.
22. A stone is dropped from the top of a building. How does its de Broglie wavelength change?
23. What do you mean by ionization energy and ionization potential?
24. Compare the wavelengths of an electron with that of a proton if their kinetic energies are equal. Mass of a proton is nearly equal to 1840 times the mass of an electron.
25. When x-rays are produced only about 10% of the initial input energy appears as x-ray energy. Explain what has happened to the other 90% of the energy..
26. What are the differences between X-rays and the ordinary ray of light?
27. What are the differences between matter wave and electromagnetic wave?
28. Why is the production of x-rays called inverse of photoelectric effect?
29. If matter has a wave nature, why is this not observable in our daily experiences?
30. Can aluminum be used as a target in X-ray tube?
31. Why a glowing gas, such as that in a neon tube, gives only certain wavelengths of light?
32. Differentiate between stimulated and spontaneous emission of radiations.
33. What do you mean by uncertainty principle?
34. An electron is in the third excited state. How many different photon-wave lengths are possible?

35. In the production of X-ray, how will you control the penetrating power of X-rays?
36. How Paschen series is originated in Hydrogen spectra?
37. Differentiate between excitation potential and ionization potential.

Long Answer Questions

38. Stating the Bohr's postulates, deduce an expression for the total energy of an electron in n^{th} orbit of hydrogen atom.
39. Describe Coolidge tube for the production of X-rays. How do you control (i)the intensity
(ii) the penetrating power of the emitted X-rays?
40. Explain the working principle of a gas laser. How is population inversion achieved for lasing action?
41. Write down the postulates of Bohr's hydrogen atom. Hence derive expression for energy of the third electron orbit.
42. State Bohr's postulates of hydrogen atom and use them to calculate the radius of n^{th} orbit of the hydrogen atom.
43. Derive Bragg's equation and explain how this equation is used to determine the crystal plane spacing.
44. State and explain uncertainty principle.

NUMERICAL PROBLEMS

45. Obtain the de Broglie wavelength of a neutron of Kinetic energy 150 ev. (Given mass of neutron = 1.675×10^{-27} kg) Planck's constant = 6.6×10^{-34} Js. 1eV = 1.6×10^{-19}J.)

Ans: 2.33×10^{-12}m

46. An x-ray tube, operated at a dc potential difference of 10 KV, produces heat at the target at the rate of 720 watt. Assuming 0.5% of the energy of incident electrons is converted into x-radiation, calculate the tube current and velocity of the incident electrons. (given, e/m = 1.8×10^{11} Ckg^{-1})

Ans: 0.072A, 6×10^{7}m/s

47. Calculate the wavelength of an electron which has been accelerated through a potential difference of 200 V. Take mass of the electronm as 9.1×10^{-31} kg and Plank's constant as 6.6×10^{-34} JS.

Ans: 8.6×10^{-11}m

48. A hydrogen atom is in ground state. What is the quantum number to which it will be excited absorbing a photon of energy 12.75eV?

Ans: n = 4, 3rd excited stage

49. Determine the wavelength of a proton that has been accelerated through a potential differences of 20 kV. Mass of proton = 1.67×10^{-27} kg and Planck constant = 6.62×10^{-34} Js.

Ans: 2.02×10^{-13} m

50. The first member of Balmer series of hydrogen atom has a wavelength of 6563 Å. Calculate the wavelength of its second member.

Ans: 4861.48 Å

51. Calculate de Broglie wavelength of an electron which has been accelerated through a potential difference of 200V. Given-mass of electron = $9.1\text{x}10^{-31}$kg and Planck's constant, h=$6.6\text{x}10^{-34}$Js.

Ans: $8.7 \text{ x } 10^{-11}$ m

52. Calculate the de Broglie wavelength of electron having kinetic energy of 400 eV.

Ans: $6.13 \text{ x } 10^{-11}$ m

53. Calculate the wave length of electromagnetic radiation emitted by a hydrogen atom which undergoes a transition between energy levels of-1.36×10^{-19} J and $-5.45\text{x}10^{-19}$ J. (Given plank constant = 6.6×10^{-34} Js).

Ans: $4.84 \text{ x } 10^{-7}$ m

54. A X-ray tube works at a dc potential difference of 50 kV. Only 0.4% of the energy of the cathode rays is converted into ∝- rays and heat is generated in the target at the rate of 600 watt. Estimate the current passed into the tube and the velocity of the electrons striking the target. (Mass of electron = 9 x 10^{-31} kg, charge of electron = 1.6 x 10^{-19} c)

55. An X-ray spectrometer has a crystal of rock salt for which atomic spacing is 2.82 Å set at an angle of 14° to the beam coming from a tube operated at a constantly increasing voltage. An instense first line appears when the voltage across the tube is 9045 V. Calculate the value of Planck constant.

Ans: $6.58 \text{ x } 10^{-34}$ Js

56. An x-ray tube works at a dc potential difference of 50KV and the current through the tube is 0.5mA. Find (i) the number of electrons hitting the target per second, (ii) the energy falling on the target per second as the kinetic energy of electrons, (iii) the cut off wavelength of x-ray emitted. (The charge of electron = 1.6×10^{-19}c, velocity of light c=3 x10^{8} m/s, Plank's constant = $6.62 x 10^{-34}$Js)

Ans: (i) 3.125×10^{15} number/sec, (ii) 25 Watt, (iii) 2.48×10^{-11} m

57. Determine the energy that must be given to a hydrogen atom so that it can emit second line of Balmer series. (Rydberg's constant R = 1 .097x10^{7} m^{-1})

Ans: 2.55 eV.

58. Find the wavelength of the radiation emitted from a hydrogen atom when an electron jumps from 4^{th} orbit to the second orbit, (given $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2N^{-1} m^{-2} h = 6.62×10^{-34} Js, $m_e = 9.1 \times 10^{-31}$ kg) (where the symbols have their usual meanings.)

Ans: 4.89×10^{-7} m

59. A cricket ball is moving with a speed of 120 km/hr. What would be its de-Broglie wavelength if its mass is 400gms.

Ans: 4.96×10^{-35}m.

60. X-rays are incident on the zinc sulphide crystal of crystal spacing 3.08×10^{-8} cm such that first order reflection takes place at glancing angle 12°. Calculate the wavelength of X-rays and glancing angle for second order maximum.

Ans: 1.28×10^{-10}m., 24.55°

61. Find the wavelength of radiation emitted from hydrogen atom when an electron jumps from third orbit to second orbit. (Given: $\varepsilon_0 = 8.854 \times 10^{-12}$ $N^{-1}m^{-2}$, h = 6.62×10^{-34} Js and mass of electron. $m_e = 9.1 \times 10^{-31}$ kg.)

Ans: 6.6×10^{-7}m

62. X-ray beam of wavelength 2.9AU (Å) is diffracted from the plane of cubic crystal. The first order diffraction is obtained at an angle of 35°. Calculate the spacing between the planes.

Ans: 5×10^{-10} m

63. X-rays are incident on the zinc sulphide crystal. It's crystal spacing is 3.08×10^{-8} cm and the first order reflection takes place at a glancing angle of 12°. Calculate the wavelength of incident X-rays.

Ans: 1.28×10^{-10}m

64. An x-ray tube operated at a d.c. potential difference of 40 KV, produces heat at the rate of 720 w assuming 0.5% of the energy of the incident electrons converted into x-radiation. Calculate (i) number of electrons per second striking the target, (ii) the velocity of the incident electrons. [Given e/m = 1.8×10^{11} C/kg).

Ans: 1.13×10^{17}, 1.18×10^{8} m/sec

65. X-rays of wavelength 0.36 Å are diffracted by a Bragg's crystal spectograph at a glancing angle of (4.8)°. Find the spacing of the atomic planes in the crystal. [4]

Ans: 2.15×10^{-10} m

66. If an electron position can be measured to an accuracy of 10^{-9} m. How accurately can its velocity be measured? ($m_e = 9.1 \times 10^{-31}$ kg.) [4]

Ans: 1.16×10^{5} ms^{-1}

67. Obtain the De Broglie wave length of the electron having the kinetic energy of 3600V.
(mass of electron = 9.1×10^{-31} kg, Electronic charge = 1.6×10^{-19}c, Plank's constant = 6.6×10^{-11} m) [4]

Ans: 2.04×10^{-11} m

68. Calculate energy in electron volts of a quantum of x-radiation of wavelength 0.15nm. Take
$e = 1.6 \times 10^{-19}$ C, $h = 6.5 \times 10^{-34}$ JS, $c = 3 \times 10^{8}$ ms^{-1}.

Ans: 8125 eV

69. An electron of energy 20eV comes into collision with a hydrogen atom in its ground state. The atom is excited into a higher state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with the emission of photon of wavelength 1.216×10^{-7}m. Determine the velocity of the scattered electron, (mass of electron = 9.1×10^{-31} kg).

Ans: 1.86×10^{6} ms^{-1}

70. Calculate the wavelength of the first line of the Balmer series, if the wavelength of the second line of this series is 4.86×10^{-7}m.

Ans: 6.56 x 10-7 m